Toy Editorial

Subtask 1

For the first subtask, we can do a state-based search with our state being positions of the horizontal and vertical part. For state transitions, we need to check if new state is valid:

- Squares occupied by the parts are empty.
- Both parts overlap.
- Both parts are entirely inside the maze.

There are $\mathcal{O}(H^2W^2)$ states and transitions from a single state can be constructed in $\mathcal{O}(K+L)$ time. The total time is $\mathcal{O}(H^2W^2(K+L))$.

Subtask 2

First, we wrap the maze in an outer layer of walls. This makes computing transitions simpler. As an extra bonus, we will not have to worry about the toy sticking out of the maze.

Now we make the transitions faster using prefix sums. For each square, we compute the number of free squares to the right:

$$r_{x,y} = \begin{cases} 0 & (x,y) \text{ is obstacle} \\ r_{x+1,y}+1 & \text{otherwise} \end{cases}$$

We can the compute number of squares down similarly. Using this we can check for empty squares in $\mathcal{O}(1)$, bringing total time down to $\mathcal{O}(W^2H^2)$.

Subtask 3

We can do a small improvement – we represent a state as intersection of the parts and their offsets from the intersection. This brings number of states down to $\mathcal{O}(WHKL)$ with time complexity being the same.

Subtask 4

We can notice that only the position of the intersection matters, as all valid states with the same intersection can be reached from each other. (For the transformation we only need to use squares occupied in at least one of the positions.)

The transitions are now bit more complicated as two valid intersections next to each other don't have to be reachable:

```
...
..X
X+-
.|.
```

If we do vertical transition we can try all the K paths which can the horizontal part take. (Each one is defined by its offset from the intersection.) We do not have to worry about the vertical part, as the new intersection must be occupied by the horizontal part, too.

We have now $\mathcal{O}(WH)$ states and transitions in $\mathcal{O}(K+L)$: $\mathcal{O}(WH(K+L))$ total.

Subtask 5

The only remaining part is improving the transitions. Let us consider moving the intersection from `A` to `B`:

```
...X<-A--->X.
X<----B->X...
```

(And suppose that we precompute space in each direction as described in Subtask 2.)

How much space does the horizontal part have? To the rightmost obstacle left from A and B and to the leftmost obstacle right:

$$\min(l_A, l_B) + \min(r_A, r_B) - 1$$

If this space is at least K, we can do the transition, otherwise not. Similarly for the vertical transition.

As we can now do the transitions in $\mathcal{O}(1)$, our final time complexity is therefore $\mathcal{O}(WH)$.